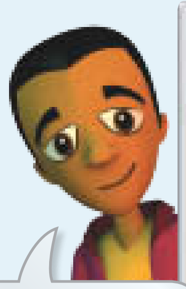
3-4 Parallel and Perpendicular Lines

Common Core State Standards

G-MG.A.3 Apply geometric methods to solve design problems.

MP 1, MP 3, MP 4

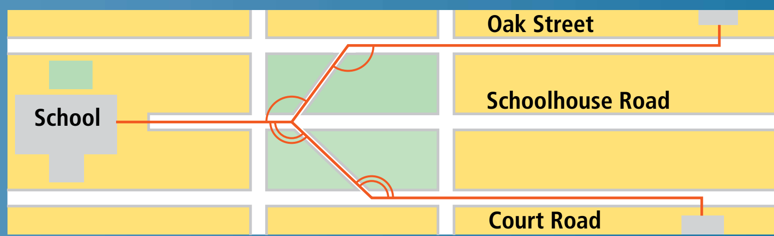
Objective To relate parallel and perpendicular lines



MATHEMATICAL PRACTICES

Getting Ready!

Jude and Jasmine leave school together to walk home. Then Jasmine cuts down a path from Schoolhouse Road to get to Oak Street and Jude cuts down another path to get to Court Road. Below is a diagram of the route each follows home. What conjecture can you make about Oak Street and Court Road? Explain.

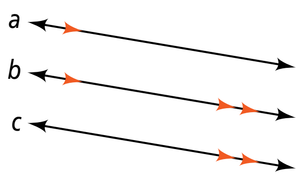


In the Solve It, you likely made your conjecture about Oak Street and Court Road based on their relationships to Schoolhouse Road. In this lesson, you will use similar reasoning to prove that lines are parallel or perpendicular.

Essential Understanding You can use the relationships of two lines to a third line to decide whether the two lines are parallel or perpendicular to each other.

take note

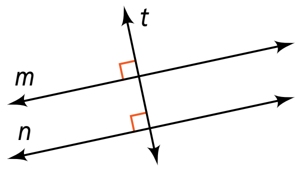
Theorem 3-8

Theorem	If . . .	Then . . .
If two lines are parallel to the same line, then they are parallel to each other.	$a \parallel b$ and $b \parallel c$ 	$a \parallel c$

You will prove Theorem 3-8 in Exercise 7.

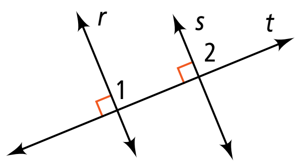
take note

Theorem 3-9

Theorem	If . . .	Then . . .
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.	$m \perp t$ and $n \perp t$ 	$m \parallel n$

Notice that Theorem 3-9 includes the phrase *in a plane*. In Exercise 17, you will consider why this phrase is necessary.

Proof Proof of Theorem 3-9



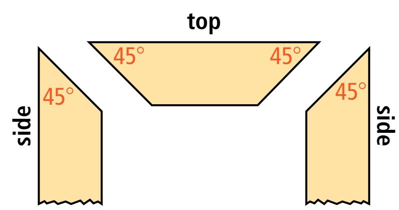
Given: In a plane, $r \perp t$ and $s \perp t$.

Prove: $r \parallel s$

Proof: $\angle 1$ and $\angle 2$ are right angles by the definition of perpendicular. So, $\angle 1 \cong \angle 2$. Since corresponding angles are congruent, $r \parallel s$.

Problem 1 Solving a Problem With Parallel Lines STEM

Carpentry **A carpenter plans to install molding on the sides and the top of a doorway. The carpenter cuts the ends of the top piece and one end of each of the side pieces at 45° angles as shown. Will the side pieces of molding be parallel? Explain.**



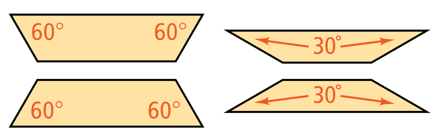
Know
The angles at the connecting ends are 45°.

Need
Determine whether the side pieces of molding are parallel.

Plan
Visualize fitting the pieces together to form new angles. Use information about the new angles to decide whether the sides are parallel.

Yes, the sides are parallel. When the pieces fit together, they form $45° + 45°$, or $90°$, angles. So, each side is perpendicular to the top. If two lines (the sides) are perpendicular to the same line (the top), then they are parallel to each other.

Got It? **1.** Can you assemble the pieces at the right to form a picture frame with opposite sides parallel? Explain.

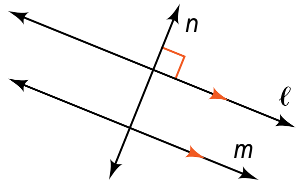


Theorems 3-8 and 3-9 give conditions that allow you to conclude that lines are parallel. The Perpendicular Transversal Theorem below provides a way for you to conclude that lines are perpendicular.

take note

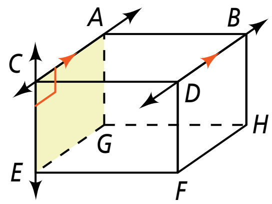
Theorem 3-10 Perpendicular Transversal Theorem

Theorem	If . . .	Then . . .
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.	$n \perp \ell$ and $\ell \parallel m$	$n \perp m$



You will prove Theorem 3-10 in Exercise 10.

The Perpendicular Transversal Theorem states that the lines must be *in a plane*. The diagram at the right shows why. In the rectangular solid, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are parallel. $\overleftrightarrow{EC}$ is perpendicular to $\overleftrightarrow{AC}$, but it is not perpendicular to $\overleftrightarrow{BD}$. In fact, $\overleftrightarrow{EC}$ and $\overleftrightarrow{BD}$ are skew because they are not in the same plane.



Proof

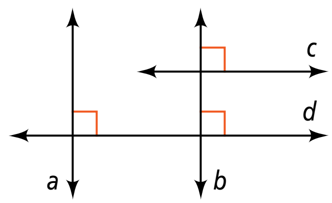
Problem 2 Proving a Relationship Between Two Lines

Think
Which line has a relationship to both lines *c* and *d*?
You know $c \perp b$ and $b \perp d$. So, line b relates to both lines c and d.

Given: In a plane, $c \perp b$, $b \perp d$, and $d \perp a$.

Prove: $c \perp a$

Proof: Lines c and d are both perpendicular to line b, so $c \parallel d$ because two lines perpendicular to the same line are parallel. It is given that $d \perp a$. Therefore, $c \perp a$ because a line that is perpendicular to one of two parallel lines is also perpendicular to the other (Perpendicular Transversal Theorem).



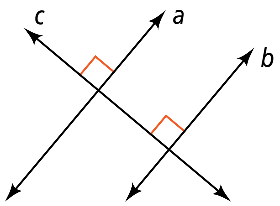
Got It? **2.** In Problem 2, could you also conclude $a \parallel b$? Explain.

Lesson Check

Do you know HOW?

1. Main Street intersects Avenue A and Avenue B. Avenue A is parallel to Avenue B. Avenue A is also perpendicular to Main Street. How are Avenue B and Main Street related? Explain.

2. In the diagram below, lines a, b, and c are coplanar. What conclusion can you make about lines a and b? Explain.

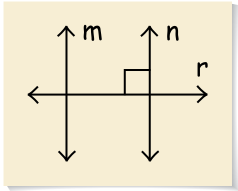


Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. Explain why the phrase *in a plane* is not necessary in Theorem 3-8.

4. Which theorem or postulate from earlier in the chapter supports the conclusion in Theorem 3-9? In the Perpendicular Transversal Theorem? Explain.

5. **Error Analysis** Shiro sketched coplanar lines m, n, and r on his homework paper. He claims that it shows that lines m and n are parallel. What other information do you need about line r in order for Shiro's claim to be true? Explain.

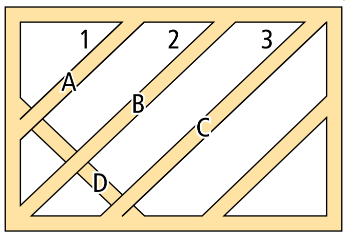


Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

See Problem 1.

6. A carpenter is building a trellis for vines to grow on. The completed trellis will have two sets of diagonal pieces of wood that overlap each other.
   a. If pieces A, B, and C must be parallel, what must be true of $\angle 1$, $\angle 2$, and $\angle 3$?
   b. The carpenter attaches piece D so that it is perpendicular to piece A. If your answer to part (a) is true, is piece D perpendicular to pieces B and C? Justify your answer.



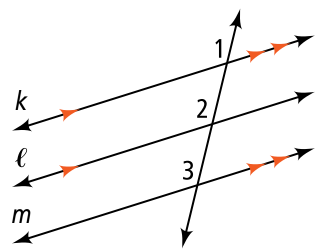
See Problem 2.

7. **Developing Proof** Copy and complete this paragraph proof of Theorem 3-8 for three coplanar lines.

Given: $\ell \parallel k$ and $m \parallel k$

Prove: $\ell \parallel m$

Proof: Since $\ell \parallel k$, $\angle 2 \cong \angle 1$ by the **a.** _?_ Theorem. Since $m \parallel k$, **b.** _?_ $\cong$ **c.** _?_ for the same reason. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. By the **d.** _?_ Theorem, $\ell \parallel m$.

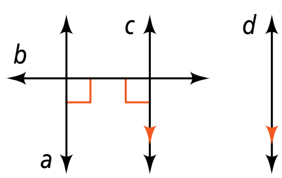


8. Write a paragraph proof.

Proof

Given: In a plane, $a \perp b$, $b \perp c$, and $c \parallel d$.

Prove: $a \parallel d$



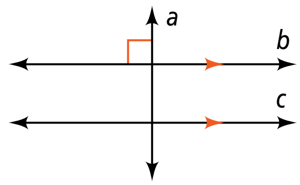
B Apply

9. **Think About a Plan** One traditional type of log cabin is a single rectangular room. Suppose you begin building a log cabin by placing four logs in the shape of a rectangle. What should you measure to guarantee that the logs on opposite walls are parallel? Explain.
   - What type of information do you need to prove lines parallel?
   - How can you use a diagram to help you?
   - What do you know about the angles of the geometric shape?

10. **Proof** Prove the Perpendicular Transversal Theorem (Theorem 3-10): In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

    Given: In a plane, $a \perp b$ and $b \parallel c$.

    Prove: $a \perp c$



The following statements describe a ladder. Based only on the statement, make a conclusion about the rungs, one side, or both sides of the ladder. Explain.

11. The rungs are each perpendicular to one side.

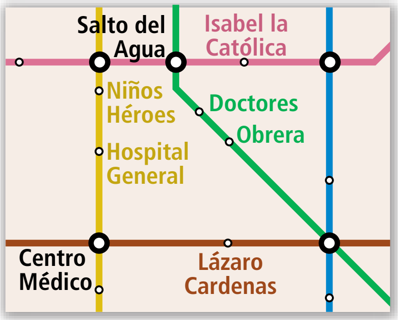
12. The rungs are parallel and the top rung is perpendicular to one side.

13. The sides are parallel. The rungs are perpendicular to one side.

14. Each side is perpendicular to the top rung.

15. The rungs are perpendicular to one side. The sides are not parallel.

16. **Public Transportation** The map at the right is a section of a subway map. The yellow line is perpendicular to the brown line, the brown line is perpendicular to the blue line, and the blue line is perpendicular to the pink line. What conclusion can you make about the yellow line and the pink line? Explain.



17. **Writing** Theorem 3-8 states that in a plane, two lines perpendicular to the same line are parallel. Explain why the phrase *in a plane* is needed. (*Hint:* Refer to a rectangular solid to help you visualize the situation.)

18. **Quilting** You plan to sew two triangles of fabric together to make a square for a quilting project. The triangles are both right triangles and have the same side and angle measures. What must also be true about the triangles in order to guarantee that the opposite sides of the fabric square are parallel? Explain.

C Challenge

For Exercises 19–24, *a*, *b*, *c*, and *d* are distinct lines in the same plane. For each combination of relationships, tell how *a* and *d* relate. Justify your answer.

19. $a \parallel b, b \parallel c, c \parallel d$

20. $a \parallel b, b \parallel c, c \perp d$

21. $a \parallel b, b \perp c, c \parallel d$

22. $a \perp b, b \parallel c, c \parallel d$

23. $a \parallel b, b \perp c, c \perp d$

24. $a \perp b, b \parallel c, c \perp d$

25. Reasoning Review the reflexive, symmetric, and transitive properties for congruence in Lesson 2-5. Write reflexive, symmetric, and transitive statements for "is parallel to" ($\parallel$). Tell whether each statement is *true* or *false*. Justify your answer.

26. Reasoning Repeat Exercise 25 for "is perpendicular to" ($\perp$).

Standardized Test Prep

SAT/ACT

27. In a plane, line e is parallel to line f, line f is parallel to line g, and line h is perpendicular to line e. Which of the following MUST be true?

(A) $e \parallel g$ (B) $h \parallel f$ (C) $g \parallel h$ (D) $e \parallel h$

28. Which point lies nearest to (5, 2) in the coordinate plane?

(F) $(-1, 3)$ (G) $(0, -2)$ (H) $(4, -5)$ (I) $(4, 10)$

29. Which of the following is NOT a reason for proving two lines parallel.

(A) The lines are both $\perp$ to the same line.

(B) Corresponding angles are congruent.

(C) Vertical angles are congruent.

(D) The lines are both $\parallel$ to the same line.

Short Response

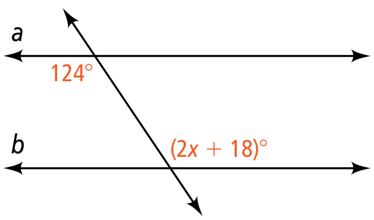
30. The diameter of a circle is the same length as the side of a square. The perimeter of the square is 16 cm. Find the diameter of the circle. Then find the circumference of the circle in terms of π.

Mixed Review

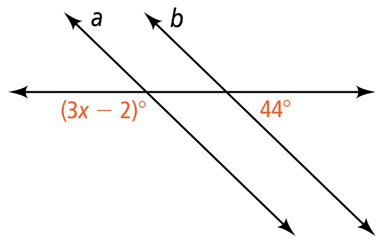
Algebra **Determine the value of x for which $a \parallel b$.**

See Lesson 3-3.

31.



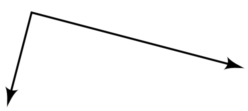
32.



Use a protractor. Classify each angle as *acute*, *right*, or *obtuse*.

See Lesson 1-4.

33.



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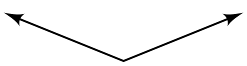
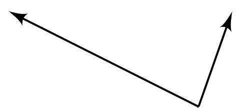


35.



Get Ready! **To prepare for Lesson 3-5, do Exercises 36–39.**

Solve each equation.

See p. 886.

36. $30 + 90 + x = 180$

37. $55 + x + 105 = 180$

38. $x + 50 = 90$

39. $32 + x = 90$